Papers written by Australian Maths Software

SEMESTER ONE

MATHEMATICS SPECIALIST

REVISION 3

UNIT 3

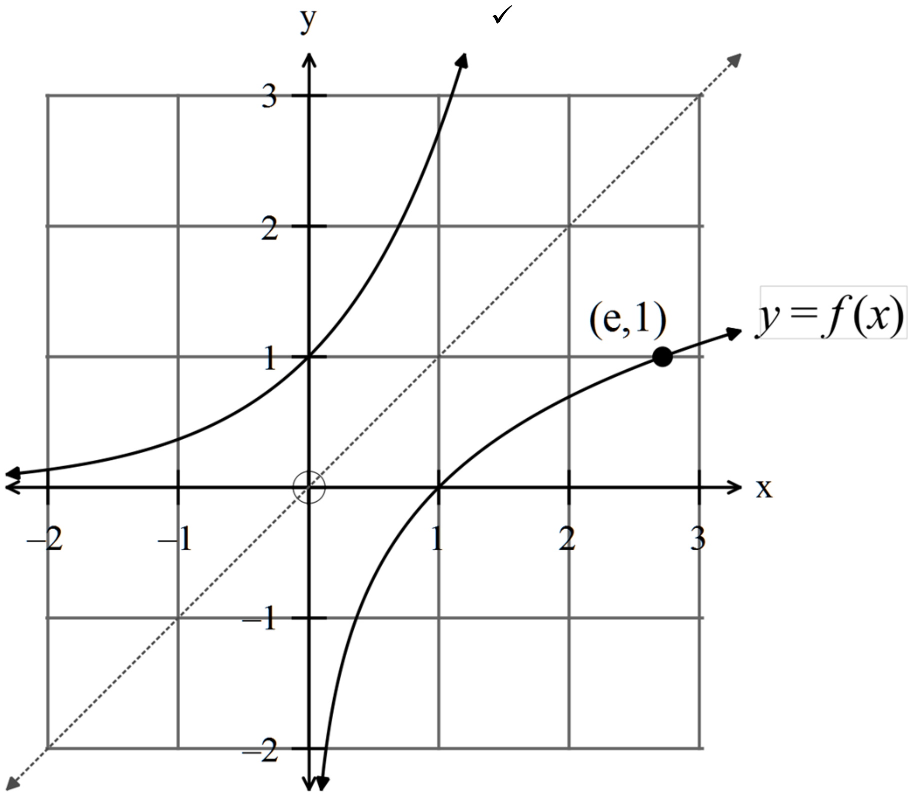
2016

SOLUTIONS

Section One

1. (6 marks)

(a) (i)



(1)

(ii) $f(x) = ln(x)$ ✓

$f^{-1}(x) = e^x$ ✓ (2)

(iii) $f^{-1}:\ x \in R$ ✓ $y > 0, y \in R$ ✓ (2)

(iv) If $f(e^2) = 2$ then $f^{-1}(2) = e^2$ ✓ (1)

2. (10 marks)

(a) $\left(\begin{array}{ccc|c} 2 & 0 & 0 & 2 \\ 0 & 5 & 0 & 10 \\ 0 & 0 & 1 & -3 \end{array}\right)$

$z=-3$

$5y=10 \Rightarrow y=2$

$2x=2 \Rightarrow x=1$

Point of intersection is $(1,2,-3)$ ✓

(1)

(b) $\left(\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right)$

$0=2$ No solution ✓

The lines of intersection of any two of the planes are parallel. ✓ (2)

(c) $\left(\begin{array}{ccc|c} 1 & 2 & -5 & -2 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$

0 = 0 An infinite number of solutions. ✓

Two of the planes could be identical and intersect the third plane or the three planes intersect in one line. ✓ (2)

(d)

$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & -1 & 2 & -6 \\ 2 & -2 & 1 & -5 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 3 & 0 & 6 \\ 0 & -4 & -1 & -5 \end{bmatrix}$ $2R_1-R_2$ ✓ R_3-2R_1 ✓

$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & -4 & -1 & -5 \\ 0 & 3 & 0 & 6 \end{bmatrix}$ Swap rows 2 and 3 ✓

$3y=6 \rightarrow y=2$

$-4(2)-z=-5 \rightarrow z=-3$

$x+2-3=0 \rightarrow x=1$ ✓

The point of intersection is $(1,2,-3)$ ✓

(5)

3. (8 marks)

(a) (i) $f(x) = -\dfrac{(x-1)}{x(x-2)}$ ✓✓ (2)

(ii) $f(x) = \dfrac{(2x-1)}{(2x)(2x-2)} = \dfrac{(2x-1)}{4x(x-1)}$ ✓✓✓ (3)

(iii) $f(x) = \dfrac{x(x-2)}{(x-1)}$ ✓✓✓ (3)

4. (13 marks)

(a) $(z-(1+2i))(z-(1-2i))(z-4)=0$ ✓

$((z-1)-2i)((z-1)+2i)(z-4)=0$

$(z^2-2z+1-4i^2)(z-4)=0$

$(z^2-2z+5)(z-4)=0$

$z^3-6z^2+13z-20=0$ ✓

$\therefore a=-6,\ b=13,\ c=-20$ ✓ (3)

(b) $z^3+2z^2+2z+1=0$

Using synthetic division with $z=-1$ You can use long division but slower

$z^3+2z^2+2z+1=0$

$$\begin{array}{r|rrrr} -1 & 1 & 2 & 2 & 1 \\ & \downarrow & -1 & -1 & -1 \\ \hline & 1 & 1 & 1 & 0 \end{array}$$

✓ method

$\therefore\ z=-1$ OR $z^2+z+1=0$ ✓

$z=\dfrac{-1\pm\sqrt{1-4}}{2}$

$z=\dfrac{-1\pm\sqrt{-3}}{2}=\dfrac{-1\pm\sqrt{3i^2}}{2}$ ✓

$z=\dfrac{-1\pm i\sqrt{3}}{2}$ ✓

$\therefore z=-1$ so $z+1$ is a factor

$$\begin{array}{r} z^2+\ z+\ 1 \\ z+1\overline{)\,z^3+2z^2+2z+1} \\ -(z^3+z^2) \\ \hline z^2+2z \\ -(z^2+\ z) \\ \hline z+1 \\ -(z+1) \\ \hline 0 \end{array}$$

$z=-1$ or $z^2+z+1=0$

(4)

(c) (i) $z^3 = -8$

$z^3 = 8\,cis(\pi)$

$z^3 = 8\,cis(\pi + 2n\pi) \quad n \in R$

$z = 2\left(cis(\pi + 2n\pi)\right)^{\frac{1}{3}}$

$z = 2cis\left(\dfrac{\pi + 2n\pi}{3}\right)$ ✓

$n = 0, \quad z = 2cis\left(\dfrac{\pi}{3}\right) = 1 + \sqrt{3}i$

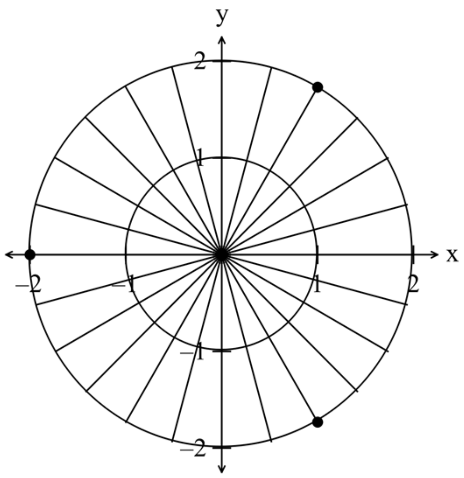
$n = 1, \quad z = 2cis\left(\dfrac{3\pi}{3}\right) = 2cis(\pi) = -1 + \sqrt{3}i$

$n = -1, \quad z = 2cis\left(-\dfrac{\pi}{3}\right) = 1 - \sqrt{3}i$ ✓✓ -1/error

(3)

(ii) The roots are $\frac{2\pi}{3}$ apart around the origin. ✓ (1)

(iii)



✓✓ -1 per error

(2)

5. (13 marks)

(a) $\left(2-2i\right)^4 = 2^4\left(1-i\right)^4$ (1)

$$= 16\left(\sqrt{2}\,cis\left(-\frac{\pi}{4}\right)\right)^4$$

$$= 16 \times 4 \times cis\left(-\pi\right)$$

$$= 64\left(-1+0i\right)$$

$$= -64 \quad \checkmark$$

(b) $\frac{3-i}{3+i} - \frac{2+i}{1-i} + \frac{1}{i}$

$$= \frac{3-i}{3+i} \times \frac{3-i}{3-i} - \frac{2+i}{1-i} \times \frac{1+i}{1+i} + \frac{1}{i} \times \frac{i}{i} \quad \checkmark$$

$$= \frac{8-6i}{10} - \frac{1+3i}{2} - i \quad \checkmark$$

$$= \frac{3-31i}{10} \quad \checkmark$$

(3)

(c) $x + yi = \left(2+3i\right)\left(3-4i\right)$

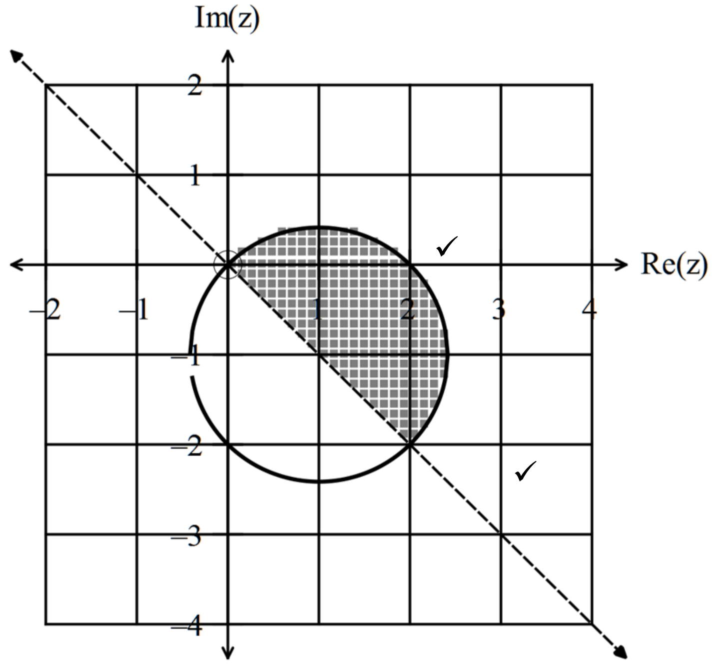
$$= 6 + 9i - 8i - 12i^2$$

$$= 18 + i$$

$x = 18, \quad y = 1$

✓ ✓ (2)

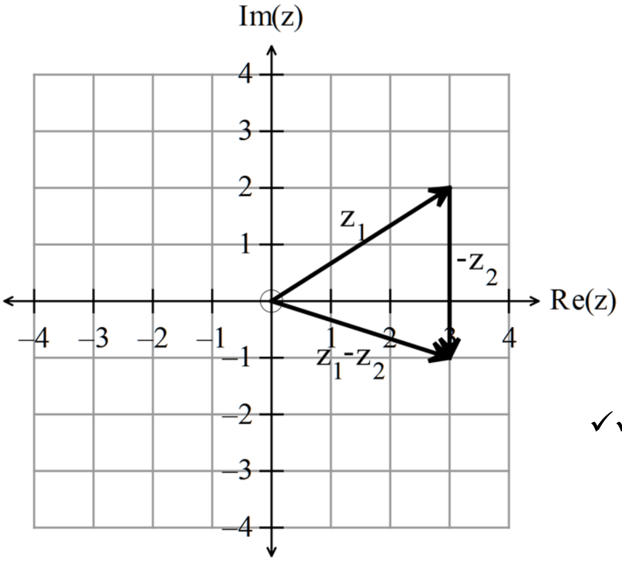
(d)



-1 per error

(2)

(e)



✓✓

(2)

(f) $$\frac{(z_1)^4}{(z_2)^3}\times z_3 = \frac{\left(cis\left(\frac{\pi}{4}\right)\right)^4}{\left(cis\left(\frac{\pi}{3}\right)\right)^3}\times\sqrt{2}\,cis\left(-\frac{\pi}{4}\right)$$

$$=\sqrt{2}\times\frac{cis\left(\frac{4\pi}{4}\right)}{cis\left(\frac{3\pi}{3}\right)}\times cis\left(-\frac{\pi}{4}\right) \quad ✓$$

$$=\sqrt{2}\times cis\left(\pi-\frac{\pi}{4}-\pi\right)$$

$$=\sqrt{2}\times cis\left(-\frac{\pi}{4}\right) \quad ✓$$

$$=\sqrt{2}\times\left(\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}}\right)$$

$$=1-i \quad ✓$$

$$\frac{(z_1)^4}{(z_2)^3}\times z_3 = 1-i$$

(3)

END OF SECTION ONE

Section Two

6. (19 marks)

(a) (i) $\mathbf{r}_1(t) = (sin(t))\mathbf{i} + (cos(t))\mathbf{j}$ and $\mathbf{r}_2(t) = (sin(t))\mathbf{i} - (cos(t))\mathbf{j}$.

$x = sin(t)\ \ y = cos(t)$ $\qquad$ $x = sin(t)\ \ y = -cos(t)$ ✓

$sin^2(t) + cos^2(t) = 1$ ✓

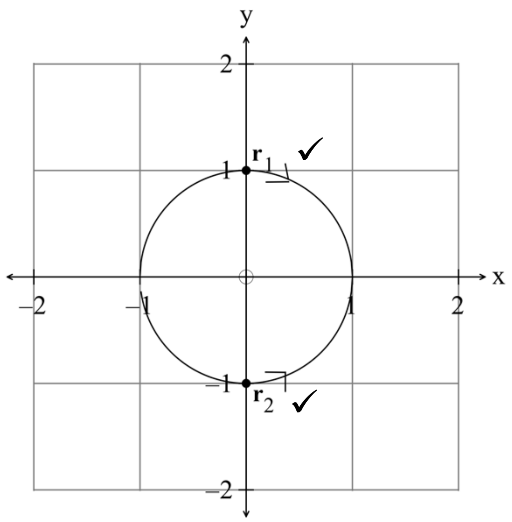
$\Rightarrow x^2 + y^2 = 1$ ✓ $\qquad$ $x^2 + y^2 = sin^2(t) + (-cos(t))^2$

$= sin^2(t) + cos^2(t)$ ✓

$\therefore x^2 + y^2 = 1$

(5)

(ii)



✓✓

(2)

(iii) $\mathbf{r}_1(0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\qquad$ $\mathbf{r}_2(0) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$ $\qquad$ See diagram $\qquad$ (2)

$\mathbf{r}_1(0^+) = \begin{pmatrix} 0^+ \\ 1^- \end{pmatrix}$ $\qquad$ $\mathbf{r}_2(0^+) = \begin{pmatrix} 0^+ \\ -1^- \end{pmatrix}$

(iv) $\mathbf{r}_1\left(\frac{\pi}{3}\right) = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}$ $\qquad$ $\mathbf{r}_2\left(-\frac{\pi}{3}\right) = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ -\frac{1}{2} \end{pmatrix}$ ✓

Distance apart is 1 unit. ✓

(2)

(b) (i) $\boldsymbol{r}(t)=\left(6\sin(t)+2\cos(t)\right)\boldsymbol{i}+\left(6\sin(t)-2\cos(t)\right)\boldsymbol{j}$.

$\boldsymbol{v}(t)=\left(6\cos(t)-2\sin(t)\right)\boldsymbol{i}+\left(6\cos(t)+2\sin(t)\right)\boldsymbol{j}$ ✓✓

$\boldsymbol{a}(t)=\left(-6\sin(t)-2\cos(t)\right)\boldsymbol{i}+\left(-6\sin(t)+2\cos(t)\right)\boldsymbol{j}$ ✓

$\boldsymbol{a}(t)=-\left(\left(6\sin(t)+2\cos(t)\right)\boldsymbol{i}+\left(6\sin(t)-2\cos(t)\right)\boldsymbol{j}\right)$ ✓

$\boldsymbol{a}(t)=-\boldsymbol{r}(t)$

(4)

(ii)

$$\boldsymbol{r}(t)\bullet\boldsymbol{v}(t)=\begin{pmatrix}6\sin(t)+2\cos(t)\\6\sin(t)-2\cos(t)\end{pmatrix}\bullet\begin{pmatrix}6\cos(t)-2\sin(t)\\6\cos(t)+2\sin(t)\end{pmatrix}$$ ✓

$$=\left(6\sin(t)+2\cos(t)\right)\times\left(6\cos(t)-2\sin(t)\right)+\left(6\sin(t)-2\cos(t)\right)\times\left(6\cos(t)+2\sin(t)\right)$$ ✓

$$=36\sin(t)\cos(t)+\cancel{12\cos^2(t)}-\cancel{12\sin^2(t)}-4\sin(t)\cos(t)+36\sin(t)\cos(t)-\cancel{12\cos^2(t)}+\cancel{12\sin^2(t)}-4\sin(t)\cos(t)$$

$\boldsymbol{r}(t)\bullet\boldsymbol{v}(t)=64\sin(t)\cos(t)$ ✓

(3)

(iii) If $\boldsymbol{r}(t)\bullet\boldsymbol{v}(t)=0$ then $64\sin(t)\cos(t)=0$

$32\sin(2t)=0$ ✓

$2t=0,\ \pi,\ 2\pi$

$t=0,\ \frac{\pi}{2},\ \pi$ ✓

(3)

7. (7 marks)

(a) $\boldsymbol{r}(t)=\int\left(\cos(5t)\boldsymbol{i}+\sin(5t)\boldsymbol{j}\right)dt$

$=\frac{\sin(5t)}{5}\mathbf{i}-\frac{\cos(5t)}{5}\mathbf{j}+\mathbf{c}$ ✓ But $\mathbf{r}\left(\frac{\pi}{5}\right)=-\frac{1}{5}\mathbf{j}$

$\therefore -\frac{1}{5}\mathbf{j}=\cancel{\frac{\sin(\pi)}{5}}\mathbf{i}-\frac{\cos(\pi)}{5}\mathbf{j}+\mathbf{c}$ ✓

$-\frac{1}{5}\mathbf{j}=\frac{1}{5}\mathbf{j}+\mathbf{c} \quad \Rightarrow \quad \mathbf{c}=-\frac{2}{5}\mathbf{j}$ ✓

$\mathbf{r}(t)=\frac{\sin(5t)}{5}\mathbf{i}-\frac{2+\cos(5t)}{5}\mathbf{j}$

(3)

(b) $\boldsymbol{r}(t) = \int \left(cos(5t)\boldsymbol{i} + sin(5t)\boldsymbol{j} \right) dt$

$\boldsymbol{v}(t) = cos(5t)\boldsymbol{i} + sin(5t)\boldsymbol{j}$ ✓

$\boldsymbol{a}(t) = -5sin(5t)\boldsymbol{i} + 5cos(5t)\boldsymbol{j}$ ✓

(2)

(c) $\mathbf{r}(t) = \frac{sin(5t)}{5}\mathbf{i} - \frac{cos(5t)}{5}\mathbf{j} - \frac{2}{5}\mathbf{j} \Rightarrow \mathbf{r}(0) = -\frac{3}{5}\mathbf{j}$ ✓

$\mathbf{v}(t) = cos(5t)\mathbf{i} + sin(5t)\mathbf{j} \Rightarrow \mathbf{v}(0) = \mathbf{i}$ ✓

(2)

8. (3 marks)

$\mathbf{AB} = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}, \ \mathbf{AC} = \begin{pmatrix} -3 \\ -1 \\ 0 \end{pmatrix}$ ✓

$\mathbf{AB} \times \mathbf{AC} = 0\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}$

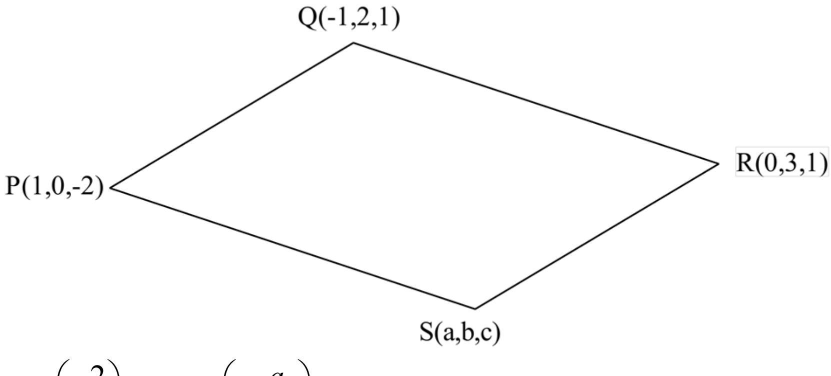
$= \begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$ ✓

Therefore the unit vector required is $\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$ ✓ NB $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ is also OK

(3)

9. (6 marks)

(a) $P(1, 0, -2), Q(-1, 2, 1)$ and $R(0, 3, 1)$ and $S(a, b, c)$



$\boldsymbol{PQ} = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}, \ \boldsymbol{SR} = \begin{pmatrix} -a \\ 3-b \\ 1-c \end{pmatrix}$ ✓

$\boldsymbol{PQ} = \boldsymbol{SR}$

$\therefore \ a = 2, \quad 2 = 3 - b \Rightarrow b = 1, \quad 3 = 1 - c \Rightarrow c = -2$

$\therefore \ S(2, 1, -2)$ ✓✓

(3)

(b) (i) $\boldsymbol{a}\times\boldsymbol{b}=\begin{pmatrix}2\\0\\3\end{pmatrix}\times\begin{pmatrix}-1\\3\\4\end{pmatrix}$

$\boldsymbol{a}\times\boldsymbol{b}=-9\boldsymbol{i}-11\boldsymbol{j}+6\boldsymbol{k}$ ✓

(1)

(ii) $Area_{\Delta}=\frac{1}{2}a\times b\times sin(C)$

$Area_{\Delta}=\frac{1}{2}|\boldsymbol{a}\times\boldsymbol{b}|$ ✓ as $|\boldsymbol{a}\times\boldsymbol{b}|=|\boldsymbol{a}||\boldsymbol{b}|sin(\theta)$ where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

$=\frac{1}{2}\left|\begin{pmatrix}-9\\-11\\6\end{pmatrix}\right|$

$=\frac{1}{2}\sqrt{81+121+36}$

$=\frac{1}{2}\sqrt{238}$

$Area_{\Delta}=7.71\ units^2$ ✓

10. (7 marks)

(a) (i) Use $\begin{pmatrix}1\\0\\1\end{pmatrix}$ to find vector between P(1,0,1) and $A(1,2,3)$. ✓

$\boldsymbol{AP}=\begin{pmatrix}0\\2\\2\end{pmatrix}$ ✓

Equation of plane is $\boldsymbol{r}(t)=\begin{pmatrix}1\\0\\1\end{pmatrix}+t\begin{pmatrix}1\\4\\3\end{pmatrix}+s\begin{pmatrix}0\\2\\2\end{pmatrix}$ ✓

(3)

(ii) $\boldsymbol{r}(t)=\begin{pmatrix}1\\0\\1\end{pmatrix}+t\begin{pmatrix}1\\4\\3\end{pmatrix}+s\begin{pmatrix}0\\2\\2\end{pmatrix}$ Test $M(6,20,16)$

$x=1+t,\quad y=4t+2s,\quad z=1+3t+2s$

If $x=6\Rightarrow t=5$ so $y=20+2s \quad z=16+2s$

If $y=20\Rightarrow s=0 \quad\therefore\ z=16$

Yes, the point $M(6,20,16)$ belongs to the line. ✓ (1)

(b) (i) John:

$$\begin{pmatrix}3\\2\\0\end{pmatrix}+\begin{pmatrix}-1\\1\\1\end{pmatrix}t=\begin{pmatrix}0\\5\\3\end{pmatrix}$$

$3-t=0\Rightarrow t=3$

$2+t=5\quad Yes,\ t=3$

$0+t=3\quad Yes,\ t=3$

John takes 3 seconds to reach the parcel. ✓ (1)

(ii) James:

$$\begin{pmatrix}-2\\1\\0\end{pmatrix}+\begin{pmatrix}1\\2\\1.5\end{pmatrix}t=\begin{pmatrix}0\\5\\3\end{pmatrix}$$

$-2+t=0\Rightarrow t=2$

$1+2t=5\quad Yes,\ t=2$

$0+1.5t=3\quad Yes,\ t=2$

James takes 2 seconds to reach the parcel so he gets to the parcel first. ✓ (1)

(iii) John:

$$\left|\begin{pmatrix}-1\\1\\1\end{pmatrix}\right|=\sqrt{3}$$

James

$$\left|\begin{pmatrix}1\\2\\1.5\end{pmatrix}\right|=\sqrt{7.25}$$

James moves with the greater speed. ✓ (1)

11. (6 marks)

(a) $$\begin{pmatrix}2m\\m\\2\end{pmatrix}\bullet\begin{pmatrix}1\\-1\\3\end{pmatrix}=0\Rightarrow 2m-m+6=0\Rightarrow m=-6$$ ✓

✓ (2)

(b) (i) $(x-1)^2+(y+3)^2+(z-2)^2=25$

Substitute $P(3,1,1)$

$(3-1)^2+(1+3)^2+(1-2)^2=4+16+1=21<25$ ✓

The point is INSIDE the circle. ✓ (2)

(ii) $$\left|\begin{pmatrix}x-1\\y+3\\z-2\end{pmatrix}\right|=5\ or\ \left|(x-1)\boldsymbol{i}+(y+3)\boldsymbol{j}+(z-2)\boldsymbol{k}\right|=5$$ ✓✓ (2)

12. (6 marks)

(a) $\boldsymbol{v}(0) = \boldsymbol{i} \qquad \boldsymbol{r}(0) = \boldsymbol{j}$ ✓

$\boldsymbol{a}(0) = -9.8\,\boldsymbol{j}$

$\boldsymbol{v}(t) = -9.8t\,\boldsymbol{j} + \boldsymbol{c_1}$

$At\ t = 0 \quad \boldsymbol{i} = \boldsymbol{c_1}$

$\therefore\ \boldsymbol{v}(t) = \boldsymbol{i} - 9.8t\,\boldsymbol{j}$ ✓

$\boldsymbol{r}(t) = t\,\boldsymbol{i} - 4.9t^2\,\boldsymbol{j} + \boldsymbol{c}_2$

$At\ t = 0 \quad \boldsymbol{j} = \boldsymbol{c}_2$

$\therefore\ \boldsymbol{r}(t) = t\,\boldsymbol{i} + \left(1 - 4.9t^2\right)\boldsymbol{j}$ ✓

$At\ \ h = 0,\ \ x = \boldsymbol{?} \qquad h = 1 - 4.9t^2$ ✓

$t^2 = 1/4.9 \quad t > 0$

$t = 0.4517539515$

$x = t\ \ so\ \ x = 0.45\,m$ ✓

The ball hits the floor 0.45 m from the table. (5)

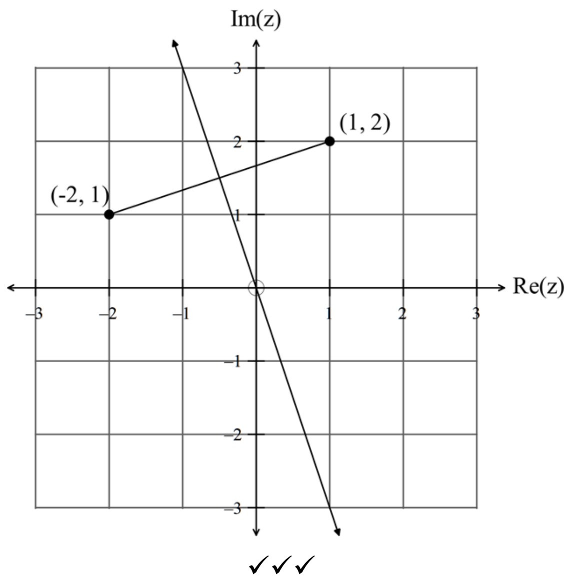
(b) The ball took 0.45 seconds to hit the floor. ✓ (1)

13. (12 marks)

(a) $\left\{ z\colon \left|z - (1+i)\right| \le 2 \ \cap\ \frac{\pi}{4} < arg(z) \le \frac{\pi}{2} \right\}$ -1/error

✓ ✓ ✓ correct inequalities (3)

(b) $|z - 1 - 2i| = |z + 2 - i|$ (1,2) (-2,1) midpoint is (-0.5,1.5)



✓✓✓ (3)

(c) (i) $\left(1+\sqrt{3}i\right)\left(1+i\right)=\left(1-\sqrt{3}\right)+\left(1+\sqrt{3}\right)i$ ✓ (1)

(ii) If $z=\left(1+\sqrt{3}i\right)\left(1+i\right)$ show that $|z|=2\sqrt{2}$ and $arg\left(z\right)=\frac{\pi}{3}+\frac{\pi}{4}$.

$$\left|\left(1+\sqrt{3}i\right)\left(1+i\right)\right|=\left|\left(1-\sqrt{3}\right)+\left(1+\sqrt{3}\right)i\right|$$

$$=\sqrt{\left(1-\sqrt{3}\right)^2+\left(1+\sqrt{3}\right)^2} \quad ✓$$

$$=\sqrt{1-2\sqrt{3}+3+1+2\sqrt{3}+3}$$

$$=\sqrt{8} \quad ✓$$

$$=2\sqrt{2}$$

$$arg\left(z\right)=arg\left(\left(1+\sqrt{3}i\right)\left(1+i\right)\right)$$

$$=arg\left(1+\sqrt{3}i\right)+arg\left(1+i\right)$$

$$arg\left(z\right)=\frac{\pi}{3}+\frac{\pi}{4} \quad ✓ \quad =\frac{7\pi}{12}$$

(3)

(iii) Show that $sin\left(\frac{7\pi}{12}\right)=\frac{1+\sqrt{3}}{2\sqrt{2}}$.

$$sin\left(\frac{7\pi}{12}\right)=\frac{Im\left(z\right)}{|r|} \quad ✓$$

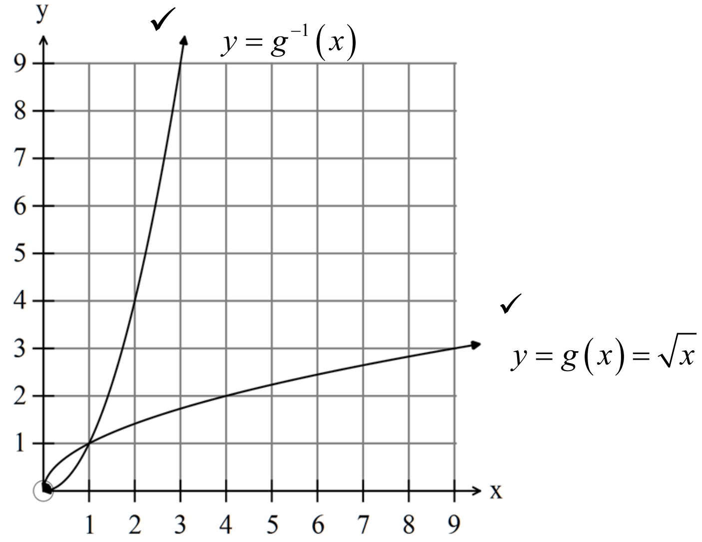
$$=\frac{1+\sqrt{3}}{2\sqrt{2}}$$ from (c) (i) / from (c) (ii) ✓

(2)

14. (17 marks)

(a) (i) The inverse exists because the function is a one to one function so for every x value, there is a unique y value. ✓✓ (2)

(ii)



(2)

(iii) $y = g^{-1}(x) = x^2 \quad x \geq 0 \qquad y = g^{-1}(x) \geq 0$ ✓✓ (2)

(iv) $g^{-1}(4) = 16$ ✓ (1)

(b) Show that $f(g(x)) = g(f(x))$.

$f(g(x)) = f(2-x) = 2(2-x) - 1 = 3 - 2x$ ✓

$g(f(x)) = g(2x-1) = 2 - (2x-1) = 3 - 2x = f(g(x))$ ✓

(2)

(c) (i) $y = p(q(x)) = p(x^2 - 3) = \sqrt{1-(x^2-3)} = \sqrt{4-x^2}$ ✓ ✓

which is defined for $-2 \leq x \leq 2$. ✓ (3)

(ii) $y = q(p(x)) = q(\sqrt{1-x}) = (\sqrt{1-x})^2 - 3 = 1 - x - 3$ ✓

$y = q(p(x)) = -x - 2 \quad$ for $x \leq 1$ ✓

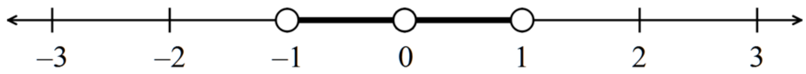
$q(p(2)) = q(\sqrt{1-2}) = q\sqrt{-1}$ which is not defined. ✓ (3)

(iii) The range of $y = q(p(x))$ is $y \geq -3$. ✓✓ (2)

15. (8 marks)

(a) (i) x = 1 or x = -1 ✓ (1)

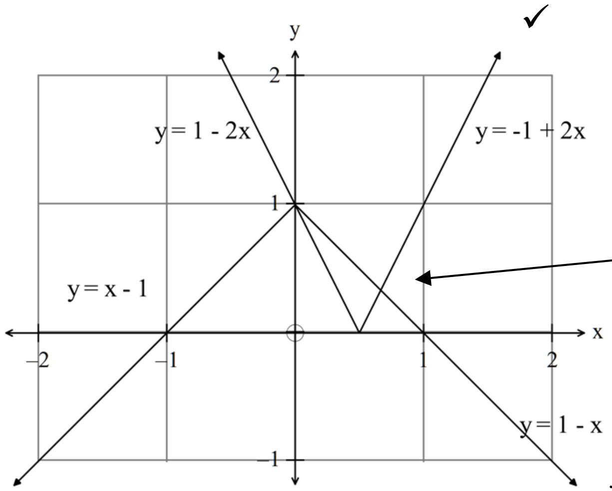
(ii) $|x| < \frac{1}{x^2}$.



✓ ✓ (2)

(b) $f(x) = 1 - |x| = \begin{cases} 1+x & \text{for } x \le 0 \\ 1-x & \text{for } x > 0 \end{cases}$ ✓

$g(x) = |1-2x| = \begin{cases} 1-2x & \text{for } x < \frac{1}{2} \\ -1+2x & \text{for } x \ge \frac{1}{2} \end{cases}$ ✓



✓

$1 - x = -1 + 2x$

$2 = 3x$

$x = \frac{2}{3}$

$x = 0 \;\; or \;\; x = \frac{2}{3}$

✓ ✓ (5)

16. (4 marks)

$$y = \frac{(x-1)(x+1)}{x(x-2)}$$ (4)

✓ ✓ ✓ ✓

17. (5 marks)

Prove that $sin(4\theta) = 4cos^3(\theta)sin(\theta) - 4cos(\theta)sin^3(\theta)$.

$sin(4\theta) = Im(cis(4\theta))$ ✓

$= Im(cis(\theta))^4$ ✓

$= Im(cos(\theta) + i\,sin(\theta))^4$ 1 4 6 4 1

$= Im\Big((cos(\theta))^4 + 4(cos(\theta))^3(i\,sin(\theta)) + 6(cos(\theta))^2(i\,sin(\theta))^2$ ✓

$+ 4(cos(\theta))(i\,sin(\theta))^3 + (i\,sin(\theta))^4 \Big)$ ✓

$= Im\left(cos^4(\theta) + 4i\,cos^3(\theta)sin(\theta) + 6i^2cos^2(\theta)sin^2(\theta) + 4i^3cos(\theta)sin^3(\theta) + i^4sin^4(\theta)\right)$

$= Im\left(cos^4(\theta) + \underline{4i\,cos^3(\theta)sin(\theta)} - 6cos^2(\theta)sin^2(\theta) - 4\underline{i\,cos(\theta)sin^3(\theta)} + sin^4(\theta)\right)$ ✓

$\therefore \quad sin(4\theta) = 4cos^3(\theta)sin(\theta) - 4cos(\theta)sin^3(\theta)$

(5)

END OF SECTION TWO